Particle reacceleration by compressible turbulence in galaxy clusters: effects of reduced mean free path

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ABSTRACT
Direct evidence for in situ particle acceleration mechanisms in the inter-galactic-medium (IGM) is provided by the diffuse Mpc–scale synchrotron emissions observed from galaxy clusters. It has been proposed that MHD turbulence, generated during cluster-cluster mergers, may be a source of particle reacceleration in the IGM. Calculations of turbulent acceleration must account self-consistently for the complex non–linear coupling between turbulent waves and particles. This has been calculated in some detail under the assumption that turbulence interacts in a collisionless way with the IGM. In this paper we explore a different picture of acceleration by compressible turbulence in galaxy clusters, where the interaction between turbulence and the IGM is mediated by plasma instabilities and maintained collisional at scales much smaller than the Coulomb mean free path. In this regime most of the energy of fast modes is channelled into the reacceleration of relativistic particles and the acceleration process approaches a universal behaviour being self-regulated by the back-reaction of the accelerated particles on turbulence itself. Assuming that relativistic protons contribute to several percent (or less) of the cluster energy, consistent with the FERMI observations of nearby clusters, we find that compressible turbulence at the level of a few percent of the thermal energy can reaccelerate relativistic electrons at GeV energies, that are necessary to explain the observed diffuse radio emission in the form of giant radio halos.

Key words: acceleration of particles - turbulence - radiation mechanisms: non–thermal - galaxies: clusters: general - radio continuum: general - X–rays: general

1 INTRODUCTION

Mergers between galaxy clusters are the most energetic events in the present Universe. During these collisions a fraction of the gravitational binding–energy of massive Dark Matter halos can be channelled into shocks and turbulence that may accelerate relativistic protons and electrons (e.g. Ryu et al 2003; Cassano & Brunetti 2005; Brunetti & Lazarian 2007; Hoeft & Brüggen 2007; Pfrommer et al 2008; Skillman et al 2008; Vazza et al 2009), while collisions between the accelerated protons and the thermal protons generate secondary particles (e.g. Blasi & Colafrancesco 1999; Pfrommer & Ensslin 2004). This makes galaxy clusters unique laboratories to study particle acceleration in diluted astrophysical plasma.

Radio observations of galaxy clusters probe these complex processes through the study of cluster–scale synchrotron emission generated by relativistic electrons that gyrate in the magnetic fields of the IGM. Giant radio halos are the most spectacular, and best studied, examples of cluster–scale synchrotron sources. They are steep–spectrum, low brightness diffuse emissions that extend similarly to the hot X–ray emitting gas (eg. Ferrari et al 2008 for a review) and that are found in merging clusters (eg. Cassano et al 2010 and ref therein). The morphological and spectral properties of a number of radio halos suggest that the emitting electrons are accelerated by spatially distributed and "gentle" (i.e. poorly efficient, with acceleration time $\sim 10^8$ yrs) mechanisms (e.g. Brunetti et al 2008).

A model put forward for the origin of giant radio halos assumes that relativistic particles in the IGM are reaccelerated by MHD turbulence that is generated during massive cluster–cluster mergers (e.g. Brunetti et al. 2001, 2004; Petrosian 2001; Fujita et al 2003; Cassano & Brunetti 2005). The theory of MHD turbulence seriously advanced in the last decades (see Cho, Lazarain & Vishniac 2003 for a review), affecting our view of particle acceleration in astrophysical plasmas (e.g. Chandran 2000; Yan & Lazarian 2002; 2004, 2008). Recently (Brunetti & Lazarian 2007, 2010) we

considered these advances to develop a comprehensive picture of turbulence in the IGM and to study stochastic reacceleration of relativistic particles. We suggested that compressible MHD turbulence (essentially fast modes), generated during energetic cluster–mergers, provides the most important driver of stochastic particle reacceleration in the IGM, and that the interaction between this turbulence and the relativistic electrons may explain the origin of radio halos, provided that a fraction (10-25 %) of the energy dissipated during mergers is channelled into these modes.

To what extend these calculations were accurate depends on our understanding of the properties of IGM turbulence. The interaction between fast modes and both the thermal IGM and the relativistic particles was assumed collisionless, in which case about 90 percent of the energy of fast modes goes into heating of the thermal plasma, while only 10 percent is available for the reacceleration of relativistic particles (eg. Brunetti & Lazarian 2007); in the following we refer to this assumption as *collisionless* IGM.

On the other hand one may think about a different picture. Indeed it can be argued that the degree of collisionality of astrophysical plasmas is underestimated when only Coloumb collisions are taken into account (eg. Lazarian et al 2010). Instabilities are naturally generated into the IGM (eg. Schekochihin et al 2005) and make the fluid more collisional, in which case a larger fraction of the energy of compressible MHD turbulence may become available for the reacceleration of relativistic particles.

At the same time the maximum energy budget available for cosmic rays in the IGM can be efficiently constrained from the recent upper limits to the gamma ray emission from nearby galaxy clusters (eg. Aharonian et al. 2009; Ackermann et al 2010) and to the Mpc-scale radio emission in clusters without radio halos (eg. Brunetti et al 2007), and this provide important information for theoretical models.

Consequently in this paper we explore the process of particle reacceleration by compressible turbulence in the IGM by assuming a picture of turbulence different from that considered in previous studies. According to this picture plasma instabilities are naturally generated in the IGM and decrease the effective collisional scale of the thermal IGM making the interaction between turbulence and the thermal plasma more collisional.

In Sect. 2 we discuss the properties of MHD turbulence and the effect of reduced mean free path in determining the way the IGM interacts with turbulence. In Sect.3 we discuss consequences of a reduced mean free path in the IGM on turbulence damping and on the reacceleration of relativistic particles; in Sect.4 we report on the case of the reacceleration of relativistic protons and of secondary electrons. In Sect.5 and 6 we provide a more general discussion and our conclusions, respectively.

2   TURBULENCE IN GALAXY CLUSTERS

2.1   Effective collisionality of the IGM

It is well known that the mean free path of thermal protons due to Coulomb collisions in the hot IGM is very large, ten to hundred kpc (e.g. Sarazin 1986). Fluids in such a collisionless regime can be very different from their collisional

counterparts (Schekochihin et al. 2005; 2010). Several instabilities (e.g. firehose, mirror, gyroresonance etc.) can be generated in the IGM in the presence of turbulence, leading to a transfer of the energy of large-scale compressions to perturbations on smaller scales.

Many instabilities have growth rate which peaks at scales near the particle gyroradius, making very large the scale separation between the energy injection scale and the scale where this energy is being deposited. On one hand scatterings induced by instabilities dramatically *reduce the effective mean free path* of thermal ions (e.g. Schekochihin & Cowley 2006) *decreasing the effective viscosity* of the IGM, at the same time these scatterings may change the *effective collisionality* of the plasma. Indeed, the usual notion of collisions in plasmas assumes Coulomb collisions. However, charged particles can be randomized if they interact with perturbed magnetic field. If this field is a result of plasma instabilities, *the process can be viewed as the collective interaction of an individual ion with the rest of the plasma*, which is the process mediated by magnetic field. As a result, the fluid would behave as collisional on scales less that the Coulomb mean free path. This issue has been addressed in Lazarian & Beresnyak (2006) for the case of a collisionless fluid subject to the gyroresonance instability that is driven by the anisotropy of the particle distribution in the momentum space that arises from magnetic field compression; the larger the magnetic field compression, the higher the anisotropy induced and the higher is the instability growth rate. They found that the turbulent magnetic compressions on the scale of the mean free path and less are the most effective for inducing the instability[*]. As the scattering happens on magnetic perturbations induced by the instability, the mean free path of particles decreases as a result of the operation of the instability. This results in the process being self-regulating, i.e. the stronger the turbulence at the scale of injection, the smaller is the mean free path of plasma particles and the larger is the span of scales over which the fluid behaves as essentially collisional.

Given these general considerations, in the following we shall assume that the interaction between the turbulent modes and the thermal IGM is similar to that of *collisional fluids* on scales which are *less than the Coulomb mean free path* but *larger than the mean free path arising from particle scattering by magnetic perturbations driven by instabilities*; in the following we refer to this assumption as *collisional* IGM.

2.2   The turbulent picture in the IGM

At large scales turbulence in the IGM is likely super-Alfvenic, the injection velocity V_L being substantially greater that the Alfven velocity v_A, in this case turbulence in the IGM behaves as hydro–turbulence (see Lazarian 2006, Brunetti & Lazarian 2007). In the Kolmogorov cascade the turbulent velocity V_l scales as $V_L(l/L_o)^{1/3}$, and at scales less than the transitional scale, $l_A \sim L_o(V_L/v_A)^{-3}$, turbulence gets sub-Alfvenic and obeys the MHD turbulence relations (see Goldreich & Srindhar 1995, and also Lazarian &

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[*] The larger scale compressions do still induce the instability, but their effect is reduced due to their reduced ability to induce large changes of B over the time scale between scattering.

Vishniac 1999, Cho & Lazarian 2003)[†] down to collisionless scales. Note, that for the IGM the scale of the transition is $l_A \sim 0.1 - 1$ kpc (e.g. Brunetti & Lazarian 2007).

In the range of scales where the interaction between turbulence and the thermal IGM is *collisional* the most important damping of turbulent motions is due to relativistic particles. This is particularly important for fast modes (Brunetti & Lazarian 2007), while it is well known that the damping of solenoidal and slow modes components of the turbulence is much less efficient, at least at relatively large scales (e.g. Yan & Lazarian 2004). In the MHD– regime, $l \leq l_A$, MHD numerical simulations have shown that a solenoidal turbulent forcing gets the ratio between the amplitude of Alfvén $\approx \delta V_s$ and fast $\approx \delta V_c$ modes in the form (Cho & Lazarian 2003) :

$$\frac{(\delta V)_c^2}{(\delta V)_s^2} \sim \frac{(\delta V)_s v_A}{c_s^2 + v_A^2} \tag{1}$$

which essentially means that coupling between these two modes is inefficient at the sufficiently small scales where the perturbations are sub-Alfvenic. This allows us to talk about separate cascades of fast, slow and Alfven modes in agreement with the simulations which performed mode separation and studied those cascades (Cho & Lazarian 2003, Kowal & Lazarian 2010).

Consequently we can assume that in the clusters of galaxies the energy transported by the cascade of fast modes is mainly channelled into the reacceleration of relativistic particles, while the compressions of magnetic field arising from slow modes transports the energy from large to small collisionless scales, sustaining the generation of the same compressible instabilities that increase the effective collisionality of the thermal IGM at scales smaller than the Coulomb scale. In this respect, to provide a more quantitative view of our picture as an example we adopt the reference case of firehose instability (e.g. Chandrasekhar et al. 1958; Barnes 1966). The threshold condition for the instability to occur with thermal electrons is $\delta T_\parallel/T > 1/\beta_{pl}$ so that this instability is expected to naturally develop in high beta plasmas, like the IGM. The cascading slow modes compress the plasma along the field lines generating anisotropies (due to conservation of adiabatic invariant) in the phase–distribution of thermal particles and potentially may drive firehose instability in the IGM. Their magnetic–field compression factor is (Cho & Lazarian 2003):

$$\frac{\delta B}{B} \sim \frac{\delta T_\parallel}{T} \approx \frac{(\delta V_l)_s}{v_A} \approx \left(\frac{l}{L_o}\right)^{1/3} (V_L/v_A)_s \tag{2}$$

(where $(V_L/c_s)_s$ is the Mach number of slow modes) that "potentially" implies a collisional scale of thermal IGM $l_f \sim 3 \times 10^{-8} L_o$[‡] when combined with the aforementioned threshold condition for the instability to occur; this is $l_f \approx 10^{-7}$

[†] MHD turbulence theory has a long history (see Biskamp 2003) and its details are still a subject of hot debates. However, recent numerical calculations are roughly consistent with the model of strong Alfvenic turbulence in Goldreich & Sridhar (1995) (see Beresnyak & Lazarian 2009) and confirm scaling of compressible modes reported in Cho & Lazarian (2003) (see Kowal & Lazarian 2010).

[‡] This assumes that the collisional scale is maintained by the instability at the minimum scale where instability occurs.

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times the classical Coulomb mean free path of the thermal IGM (assuming $L_o \approx 200 - 300$ kpc).

3 DAMPINGS OF FAST MODES AND PARTICLE ACCELERATION

Having motivated our picture of a *collisional* IGM, in this Section we discuss consequences on reacceleration of relativistic particles by fast modes.

We consider the most simple situation where turbulence is injected at a single scale, with wavenumber k_o, and assume that a fraction of the turbulent energy–flux is channelled into fast modes, with rate $I^f(k,t) = I_o^f \delta(k - k_o)$. Under these conditions fast modes can be assumed isotropic (e.g. Cho & Lazarian 2003) with quasi–stationary spectrum (Brunetti & Lazarian 2007, 2010) :

$$\mathcal{W}^f(k) \approx c_w \left(I_o^f \rho \langle V_{\mathrm{ph}} \rangle\right)^{\frac{1}{2}} k^{-\frac{3}{2}} \tag{3}$$

for $k_o < k < k_c$, where c_w is of the order of unity and k_c is the cut–off scale where collisionless dampings become more efficient than the process of wave–wave cascading.

Under the hypothesis discussed in the previous Section, the most important collisionless damping of fast modes is due to the Transit-Time-Damping (TTD) resonance with relativistic particles (eg. Schlickeiser & Miller 1998; Yan & Lazarian 2004; Brunetti & Lazarian 2007, 2010):

$$\Gamma_{CR} \approx -\frac{\pi^2}{8} k \langle \frac{|B_k|^2}{\mathcal{W}^f} \rangle \frac{\sin^2 \theta}{|\cos \theta|} \frac{c_s^2}{B_o^2} \int p^4 dp \frac{\partial f(p)}{\partial p} \tag{4}$$

where B_o is the background (unperturbed) magnetic field, c_s is the sound speed and $|B_k|^2/\mathcal{W}^f$ is the ratio between magnetic field fluctuations and total energy in the mode (the quantity $\langle .. \rangle$ indicates average with respect to the angle between mode wavenumber and the background magnetic field). The cut–off scale is[§] :

$$k_c = k_c^{CR} = c_k \frac{I_o^f}{\rho c_s} \left(\frac{\langle \Gamma_{CR}(k,\theta) \rangle}{k}\right)^{-2} \tag{5}$$

where $c_k \sim$ a few (Brunetti & Lazarian 2007; see also Matthaeus & Zhou 1989, for details on Kraichnan constants).

In this case all the energy of fast modes is channelled into the reacceleration of relativistic particles. The particle–diffusion coefficient in the momentum space due to TTD is obtained combining e.g. Eq. 47 in Brunetti & Lazarian (2007) and Eq. 5 :
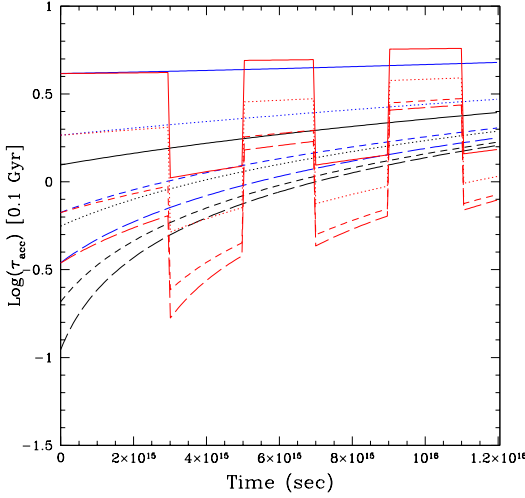
$$D_{pp} \simeq 2c_w c_k^{1/2} \frac{p^2 I_o^f}{\sum_{e,p} \int dp p^2 c \left|\frac{\partial N}{\partial p} - 2\frac{N}{p}\right|} \tag{6}$$

where, assuming $I_o^f \phi \tau >> \epsilon_{CR}^o$ (ϵ_{CR}^o the initial energy density of cosmic rays), it is[¶]:

$$\int dp p^2 c \left|\frac{\partial N}{\partial p} - 2\frac{N}{p}\right| \sim c_N \epsilon_{CR} \sim I_o^f \phi c_N \tau \tag{7}$$

[§] provided that $k_c \leq k_{coll}$, k_{coll} being the wavenumber where the IGM becomes collisionless

[¶] we assume that damping is dominated by a single species of relativistic particles

Figure 1. The systematic acceleration time–scale is reported as a function of time. The initial energy density of cosmic rays is $\epsilon_{CR}^o = 0.3$ (black lines) and 3% (blue lines) of the thermal IGM. Calculations are reported assuming $(V_L/c_s)^2$ =0.02 (long–dashed lines), 0.03 (dashed lines), 0.05 (dotted lines) and 0.07 (solid lines). Red lines show a case of intermittent turbulence where we assume $\epsilon_{CR}^o = 0.03\epsilon_{th}$ and, for simplicity, we assume that the turbulent energy varies with time $0.25 \times (V_L/c_s)^2$ and $(V_L/c_s)^2$ (same line–code given above). All calculations assume a spectral index of cosmic rays $\delta = 2.6$, a IGM number density $n_{th} = 7 \cdot 10^{-4} \mathrm{cm}^{-3}$ and temperature $T = 10^8$ K.

c_N is a numerical factor that depends on the shape of the spectrum of the accelerated particles[||]. ϕ accounts for the "intermittent" nature of the turbulence–injection process (i.e., $I_o^f \phi \tau$ is the energy density injected into fast modes in the period of time τ), we indeed expect that many patches of large–scale turbulence can be injected at different times in a Mpc^3 region during a merger.

From Eqs.6 and 7 the process of particle reacceleration is self–regulated by the damping of the modes due to the reaccelerated particles.

This is a new regime of particle reacceleration by compressible turbulence. Indeed it greatly differs from the case of a *collisionless* IGM, where the damping of the modes is dominated by the collisionless damping with the thermal electrons in the IGM (e.g. Cassano & Brunetti 2005; Brunetti & Lazarian 2007, 2010). On the other hand, a similar effect, the *proton–wave boiler*, was observed in the case of reacceleration by a hypothetical spectrum of isotropic Alfvenic waves (Brunetti et al 2004) where indeed the damping of the modes was dominated by gyro–resonance with relativistic protons. According to Eqs. 6 and 7 we can identify two asymptotic regimes of reacceleration ($\phi = 1$ for simplicity) : (i) a *fast regime* for $I_o^f \tau_f \sim \rho V_L^2 >> \epsilon_{CR}^o$, where τ_f is the cascading time of fast modes injected at large scales, and (ii) a *slow regime* for $I_o^f \tau_f << \epsilon_{CR}^o$.

In the *fast* regime the damping due to relativistic particles is

[||] for instance $c_N = (s+2)$ assuming $N(p) \propto p^{-s}$

initially small making the acceleration efficiency large. Under these conditions relativistic protons rapidly gain energy with the consequence that the damping of the turbulent modes by these protons increases with time and makes the reacceleration process less efficient. After few acceleration times, as soon as $\epsilon_{CR} \sim I_o^f \tau$, we expect that the process approaches a "asymptotic", universal, regime that does not depend on initial conditions (provided that the injection rate of turbulence is constant). In this case the particle–diffusion coefficient in the momentum space reads (from Eqs. 6 and 7):

$$D_{pp} \sim 2 \frac{c_w c_k^{1/2}}{c_N \phi} \tau^{-1} \qquad (8)$$

i.e. the reacceleration efficiency approaches a universal behaviour and decreases (linearly) with time.

On the other hand, in the *slow regime* relativistic protons do not increase significantly their energy implying a quasi–constant damping of the modes; in this case $D_{pp} \propto I_o^f$.

All these effects can be seen in Figure 1, where we report the evolution with time of the systematic reacceleration time, $\tau_{acc} = p^2/(4D_{pp})$, assuming different initial values of $I_o^f \tau_f/\epsilon_{CR}^o$ (for simplicity the thermal energy density and I_o^f are assumed constant with time). For small $\rho V_L^2/\epsilon_{CR}^o$ we are in the *slow regime* where the acceleration time does not change significantly with time. On the other hand, for large $\rho V_L^2/\epsilon_{CR}^o$, we are in the *fast regime* where initially the acceleration time rapidly increases and then approaches the "asymptotic" behaviour at later stages. By adopting a more realistic picture where both the thermal energy density of the IGM and I_o^f may increase with time during a merger, or where turbulence is intermittent (Fig. 1) we expect a less rapid approach to this "asymptotic" behaviour.

We believe that the most important consequence of this new regime of particle reacceleration by compressible turbulence is the universal acceleration time–scale, $\sim 10^8$ yrs, that occurs when *turbulence and cosmic rays reach approximate equipartition*.
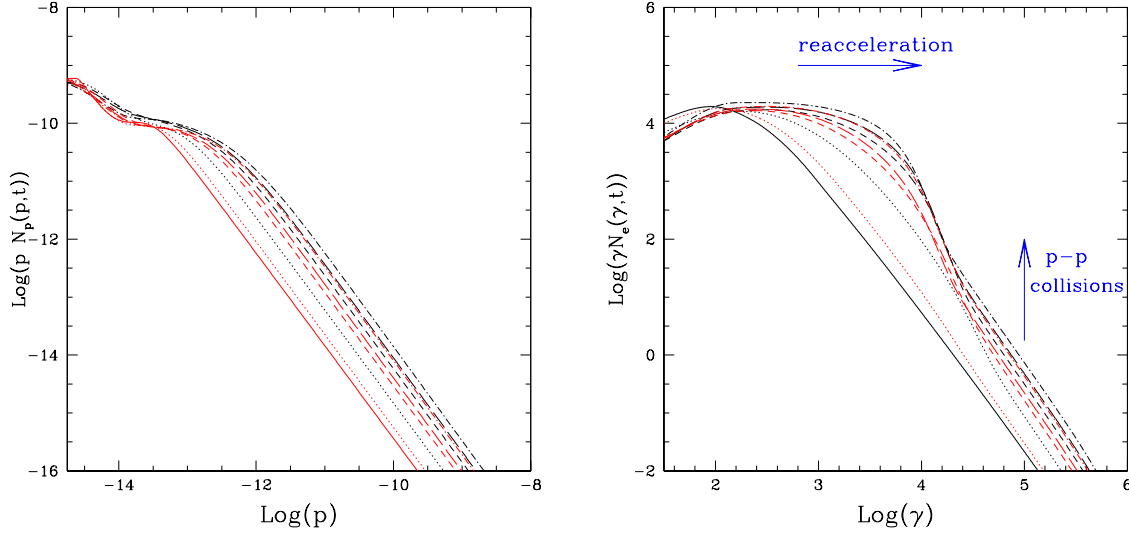
4   REACCELERATION OF PROTONS AND SECONDARY ELECTRONS

As an example we calculate the evolution with time of the spectrum of relativistic electrons and protons subject to reacceleration by fast modes assuming a *collisional* IGM. As a simplification we consider only primary protons and the secondary electrons produced by inelastic collisions between these protons and the IGM. In this case the damping of the modes is largely dominated by that with relativistic protons. This also allows for a prompt comparison with similar calculations developed under the assumption of *collisionless* IGM (Brunetti & Lazarian 2010).

We model the time evolution of the spectral energy distribution of electrons, N_e, with an isotropic Fokker-Planck equation :

$$\frac{\partial N_e(p,t)}{\partial t} = \frac{\partial}{\partial p}\left[N_e(p,t)\left(\left|\frac{dp}{dt}\right|_r - \frac{1}{p^2}\frac{\partial}{\partial p}(p^2 D_{pp})\right.\right.$$
$$\left.\left. + \left|\frac{dp}{dt}\right|_i\right)\right] + \frac{\partial^2}{\partial p^2}\left[D_{pp} N_e(p,t)\right]$$

Figure 2. The evolution with time of the spectrum of relativistic protons (**Left Panel**) and of (secondary) electrons (**Right Panel**). Results are reported for time $\tau = 0$ (solid lines), 3 (dotted lines), 9 (short–dashed lines), 12 (long–dashed lines), 16 $\times 10^{15}$sec (dot–dashed lines). Calculations are obtained assuming $\epsilon^o_{CR} = 1\%$ of the thermal energy, and $(V_L/c_s)^2 = 0.07$ (black lines) or intermittent turbulence (red lines) (according to Figure 1, with 1 and $0.25\times (V_L/c_s)^2 = 0.07$). Thermal parameters are the same of Figure 1, a magnetic field $B_o = 2.5\mu$G and redshift $=0.15$ are assumed.

$$+Q_e[p, t; N_p(p, t)] \,, \qquad (9)$$

where $|dp/dt|$ marks radiative (r) and Coulomb (i) losses exsperienced by relativistic electrons in the IGM (see Brunetti & Lazarian 2010 and ref therein for the relevant formulae), D_{pp} is the electron diffusion coefficient in the momentum space due to the coupling with fast modes (Eq.6), and the term Q_e accounts for the injection rate of secondary electrons due to p-p collisions in the IGM (following Brunetti & Lazarian 2010).

Similarly the time evolution of the spectral energy distribution of protons, N_p, is given by :

$$\frac{\partial N_p(p, t)}{\partial t} = \frac{\partial}{\partial p}\left[N_p(p, t)\left(\left|\frac{dp}{dt}\right|_i - \frac{1}{p^2}\frac{\partial}{\partial p}(p^2 D_{\mathrm{pp}})\right)\right]$$
$$+ \frac{\partial^2}{\partial p^2}\left[D_{\mathrm{pp}}N_p(p, t)\right] - \frac{N_p(p, t)}{\tau_{pp}(p)} \,, \qquad (10)$$

where $|dp/dt_i|$ marks Coulomb losses and τ_{pp} is the proton life–time due to pp collisions in the IGM (see Brunetti & Lazarian 2010 and ref therein), and D_{pp} is the diffusion coefficient in the momentum space of protons due to the coupling with fast modes (Eq.6).

The spectral shape of relativistic particles and its evolution with time is expected to be similar to that in the case of turbulent reacceleration in models that assume a *collisionless* IGM. Relativistic protons do not experience relevant energy losses in the IGM and TTD resonance in the IGM may reaccelerate supra-thermal protons up to high energies. On the other hand, the case of electrons is more complex. Radiative losses (due to synchrotron and inverse Compton emission) experienced by relativistic electrons in the IGM are expected to prevent an efficient reacceleration of these

particles above a maximum energy, γ_{max}. Consequently, at higher energies the evolution of the electron spectrum is basically driven by the process of injection of fresh electrons due to p–p collisions (and on its interplay with radiative losses).

Results are reported in Figure 2. Calculations are carried out considering that cosmic ray protons contribute to a few percent of the thermal cluster energy, consistent with the recent limits derived from FERMI observations of nearby clusters (Aharonian et al. 2009; Ackermann et al 2010). Under the hypothesis of *collisional* IGM, the bulk of turbulence (fast modes) is channelled into reacceleration of cosmic rays and the FERMI limits put corresponding constraints also on the energy density of turbulent motions in the IGM, about $(V_L/c_s)^2 \leq 10\%$. Fig. 2 shows that relativistic (secondary) electrons can be reaccelerated at energies of several GeV even by assuming that fast modes contribute to only a few percent of the energy density of the IGM. On the other hand a larger amount of compressible turbulence is typically requested assuming a *collisionless* IGM (e.g. Brunetti & Lazarian 2010 and ref therein). This however does not imply that relativistic electrons can be reaccelerated for long periods at energies much larger than those in the *collisionless* case. Indeed larger injection rates of turbulence do not make the reacceleration process substantially more efficient, due to the damping by the relativistic protons that self-regulates the acceleration efficiency in a few acceleration times. The effect of damping is also visible in Figure 2 : at later stages the acceleration of cosmic rays starts to saturate.

It is important ti stress that self-regulation depends on $I^f_o \phi\tau/\epsilon_{CR}$, thus the effect of proton back–reaction on the acceleration efficiency becomes less important in the (more

realistic) case of intermittent (or patchy) turbulence ($\phi < 1$, Fig. 2) that implies that less energy in channelled into cosmic ray protons; in this case larger acceleration efficiencies are maintained for longer periods of time (see also Fig. 1).

5 DISCUSSION

Whether the thermal IGM is *collisional* or *collisionless* at scales smaller than the Coulomb scale depends on the effect of reduced mean free path that is mediated by the plasma instabilities. Consequently the way compressible turbulence is damped and particle are reaccelerated in the IGM depends on the interplay between several reference scales (wavenumbers): the collisionless scale, k_{coll}, the Coulomb scale, k_C, the turbulence cut–off scale due to collisionless damping with thermal particles, k_c^{th}, and that due to collisionless damping with relativistic particles, k_c^{CR} (Figure 3 for a sketch of both *collisional* and *collisionless* cases).

The cut–off scale due to collisionless damping with thermal particles is (Brunetti & Lazarian 2007) :

$$k_c^{th} \sim \mathcal{C}^{th} k_o \left(\frac{V_L}{c_s} \right)^4 \tag{11}$$

($\mathcal{C}^{th}$ is a constant) while that due to collisionless damping with relativistic particles (from Eqs.4 and 5) is :

$$k_c^{CR} \sim \mathcal{C}^{CR} k_o \left(\frac{V_L}{c_s} \right)^4 \left(\frac{\epsilon_{CR}}{\epsilon_{th}} c_s \right)^{-2} \tag{12}$$

($\mathcal{C}^{CR}$ is a constant) where $k_c^{CR} >> k_c^{th}$ in the IGM (Brunetti & Lazarian 2007).
The typical spatial scale of the collisionless (TTD) thermal damping is fairly large, $1/k_c^{th} \sim 0.1 - 1 \times \frac{L_o(kpc)}{300} (\frac{V_L^2/c_s^2}{0.3})^{-2}$ kpc (Brunetti & Lazarian 2007). Consequently it is reasonable to assume that plasma instabilities driven by compressible turbulence (even considering a modest level of turbulence) may maintain the IGM at least *weakly–collisionless*, with $k_c^{CR} > k_{coll} >> k_c^{th}$, allowing for an increasing fraction of turbulence to become available for reacceleration of relativistic particles with respect to the case of *collisionless* IGM. In this regime the damping of turbulence is collisional (as in Sect. 4) at scales larger than $\sim 1/k_{coll}$, whereas the cascade of fast modes is suddenly interrupted by collisionless TTD with thermal electrons as soon as $1/k \leq 1/k_{coll}$. In this regime the momentum–diffusion coefficient of relativistic particles subject to TTD resonance can be estimated as (from Eq.(47) in Brunetti & Lazarian 2007)[★★] :

$$D_{pp} \approx \frac{\pi}{8} \frac{p^2}{c} \langle \frac{\beta_{pl}|B_k|^2}{16\pi \mathcal{W}^f} \rangle \left(\frac{2I_o^f c_s}{7\rho} \right)^{\frac{1}{2}} k_{coll}^{\frac{1}{2}} \mathcal{I} \tag{13}$$

where both $\mathcal{I}$ and $\langle ... \rangle \sim$ a few.
As in Sect. 2 we consider the case of firehose instability where the collisionless scale is (from Eqs.2 and 5):

$$k_{coll} \sim 8 \, k_o f_{slow}^{3/2} \left(\frac{V_L}{c_s} \right)^3 \left(\frac{c_s}{v_A} \right)^9 \tag{14}$$

★★ Here we are assuming that the dominant damping of fast modes is still provided by the resonance with thermal particles at scales $\leq 1/k_{coll}$

where f_{slow} is the ratio of the energy of slow and fast modes. Consequently the momentum–diffusion coefficient in the *weakly–collisionless* regime can be estimated :

$$D_{pp} \sim \mathcal{C}_D \, p^2 c_s^2 k_o \left(\frac{V_L}{c_s} \right)^{\frac{7}{2}} \left(\frac{c_s}{v_A} \right)^{\frac{9}{2}} f_{slow}^{3/4} \, , \tag{15}$$

($\mathcal{C}_D$ a constant) i.e. $D_{pp} \propto T \beta_{pl}^{9/4}$ (for a fixed V_L/c_s), that has the same scaling with temperature derived in the collisionless regime.

As explained in the previous Section, an increasing fraction of turbulent energy available for the reacceleration of cosmic rays does not imply that the acceleration efficiencies are much larger than in the *collisionless* case, since if we assume that most of the turbulent energy is channelled into cosmic rays the back reaction of these particles self-regulates the acceleration process approaching a universal regime.

Non–thermal radiations from galaxy clusters are probes of in situ particle acceleration processes in the IGM that are activated in massive (hot) galaxy clusters during cluster–cluster mergers (e.g. Cassano et al 2010). Previous turbulent reacceleration models for the origin of giant radio halos were based on the assumption of a *collisionless* IGM. In the context of these models the fact that nowadays giant radio halos are found only in massive (and hot) clusters is interpreted via simple energetics arguments that stem in the expectation that the turbulent injection rate increases with the mass of the merging clusters (e.g. Cassano & Brunetti 2005).
Additional inputs may come from our explorative study. We believe that the picture may be more complex than previously thought. Assuming that the compressible turbulence is generated at large (injection) scales and then cascades to smaller scales, we suggest that the effects of reduced mean free path in a turbulent IGM may allow the *fraction* of the energy of turbulence that is available to the reacceleration of relativistic particles to be larger than that derived for a *collisionless* IGM. This readily implies that the damping from relativistic particles plays a role in regulating the acceleration efficiency and introduces a new physical threshold in the mechanism responsible for the reacceleration of the relativistic particles. The threshold is connected with the energy contributed by cosmic rays in the IGM: the level of compressible turbulence in the IGM of hot, merging, clusters may become comparable to (or larger than) that contributed by cosmic rays, while the damping due to these particles may suppress particle reacceleration in less turbulent, relaxed clusters.
On the other hand this complex picture does not affect the basic expectations from previous studies. Indeed we show that the scaling of the acceleration efficiency with IGM temperature derived assuming a *collisionless* IGM may also extend to the case of a *weakly–collisionless* IGM implying that the conclusion that stochastic acceleration is stronger in the hottest clusters holds for a wide range of (micro–)physical conditions. By considering the example where the collisionless scale is regulated by the effects of firehose instability, both the acceleration efficiency and the collisional or collisionless nature of the interaction between turbulence and the IGM depend on the beta of the plasma plays, with the IGM being more *collisional* for larger values of β_{pl} (Eq.14).

In this paper we have considered only the case of stochastic reacceleration due to compressible turbulence (es-
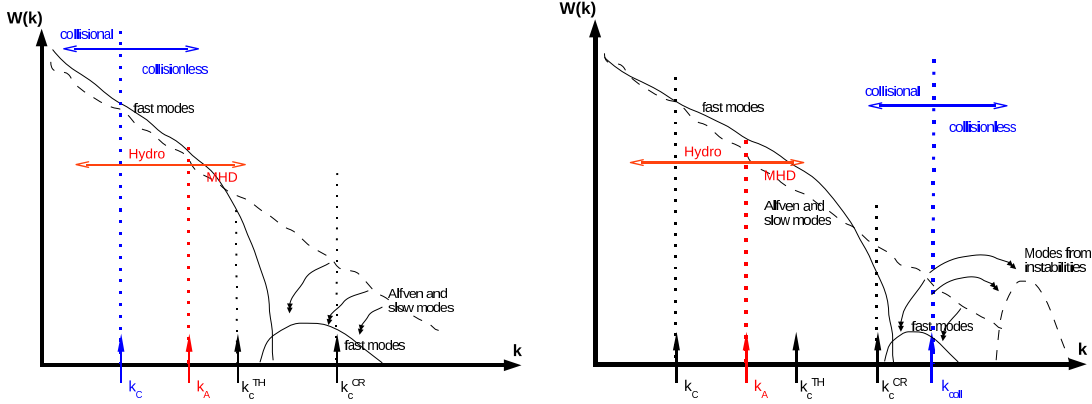
Figure 3. A cartoon giving the evolution with scale of turbulence (solid lines = fast modes, dashed lines = Alfvén and slow modes) assuming collisionless interaction with thermal IGM (e.g. Brunetti & Lazarian 2007) (**Left Panel**) and collisional interaction mediated by plasma instabilities (this paper) (**Right Panel**). In the Figure we also mark the generation (thin arrows) of fast modes from the cascading of Alfvén and slow modes (both panels) and that of solenoidal modes from plasma instabilities (right panel). The position of the relevant scales, Coulomb scale (k_C), MHD scale ($l_A^{-1} \sim k_A$), thermal and relativistic cut–off scales (k_c^{th} and k_c^{CR}, respectively) is reported, together with that of the effective collisionless scale k_{coll} (in the right panel; $k_{coll} = k_C$ in the left panel). The exponential cut–off in the spectrum of fast modes marks the effect of collisionless damping with thermal IGM (left panel) and with relativistic particles (right panel).

sentially fast modes). However we do not think that the overall picture of turbulent reacceleration described in the present paper is complete. Indeed the processes of *in situ* reacceleration of particles in the clusters of galaxies is not limited to the acceleration by fast modes. For instance, processes of magnetic reconnection in turbulent astrophysical plasmas can induce additional acceleration (see Lazarian & Opher 2009, Drake et al. 2010, Lazarian & Desiati 2010, Brunetti & Lazarian 2010 and references therein). Also Alfvén modes were proposed for the reacceleration of relativistic particles in the IGM (e.g. Fujita et al 2003, Brunetti et al 2004). For instance instabilities in cosmic ray collisionless fluid can induce small-scales Alfven waves with **k** parallel to magnetic fields and those waves can efficiently induce cosmic ray reacceleration (Lazarian & Beresnyak 2006, Brunetti & Lazarian 2010, Yan & Lazarian in prep.), a process that requires further investigation. We want to stress that overall, the existence of all these processes in a turbulent IGM is suggestive that even larger share of energy (compared to our present estimates) of turbulence energy can be transferred to relativistic particles in the galaxy clusters.

6 CONCLUSIONS

It has been proposed that the observed giant radio halos may be due to turbulent reacceleration of relativistic particles in merging clusters. Calculations of particle acceleration by MHD turbulence must account self-consistently for the non–linear interaction between turbulent waves and particles. Previous theoretical works in this context focus on the interaction between compressible turbulence (fast modes) and both the thermal IGM and the relativistic particles by assuming a *collisionless* IGM. In this case about 90 percent of the energy of fast modes goes into heating of the thermal plasma, while only 10 percent is available for the reacceleration of relativistic particles (eg. Brunetti & Lazarian 2007).

    In this paper we explore a new possibility for the par-

ticle reacceleration by compressible turbulence where the interaction between turbulent modes and the thermal IGM is *collisional* at scales much smaller than the Coulomb mean free path. We have motivated this guess by observing that several plasma instabilities can be generated by turbulent motions in the IGM driving perturbations in the magnetic field that induce scattering of charged particles on scales much smaller than the Coulomb mean free path. This process results in a *collective* interaction of individual ions with the rest of the plasma and potentially constrains the effective collisionless scale to (about) the scales where instabilities develop.

Under these conditions we find that an increasing fraction of the energy of fast modes is available to the reacceleration of relativistic particles and that the collisionless damping of turbulent motions induced by these particles contribute to self-regulate the acceleration process. Assuming a *collisional* IGM, all the energy of fast modes is channeled into the reacceleration of relativistic particles. Interestingly in this case the upper limits to the energy densities of cosmic rays in galaxy clusters derived by recent gamma and radio observations provide also constraints to the fraction of the thermal energy in the IGM available for fast modes. In a *collisional* IGM the acceleration efficiency results from the balance between the injection rate of compressible turbulent motions and the rate of turbulent damping due to the reaccelerated particles. In this case we find that a universal acceleration regime is established as soon as turbulent energy approaches equipartition with the energy density of relativistic particles, and that in this case typical (systematic) reacceleration times $\sim 10^8$ yrs are provided by TTD with fast modes.

Under the assumption of a *collisional* IGM we calculate the reacceleration of relativistic protons and of secondary relativistic electrons. Based on present constraints from FERMI observations of nearby clusters, we consider a situation where cosmic rays contribute to a few percent of the thermal

energy of the IGM and show that relativistic electrons can be reaccelerated at energies of several GeV, provided that compressible turbulence, generated at large scales, contributes to several percent of the cluster thermal energy. The reaccelerated electrons emit synchrotron radiation at GHz frequencies in typical (several) μG magnetic fields providing an explanation for the origin of the observed giant radio halos. Remarkably, in the case of *collisional* IGM the amount of cluster turbulence necessary to reaccelerate GeV electrons is significantly smaller than that in models that assume a *collisionless* IGM.

Whether the interaction between turbulence and IGM behaves *collisionless* or *collisional* depends on the effect of the reduced mean free path. It is reasonable to assume that the IGM becomes (at least) *weakly–collisionless* as soon as a sufficient (even modest) level of compressible turbulence is generated. This has important consequences on the process of particle acceleration by compressible turbulence in the IGM. All the calculations of turbulent reacceleration developed under the assumptions of a *collisionless* IGM should be retained as conservative approaches. Consequently our explorative study provides further theoretical support to the idea that turbulence may play an important role in the the origin of non-thermal components (and emission) on cluster scales.

Finally we want to remark that although we believe that fast modes may play an important role in the process of *in situ* reacceleration, additional processes in a turbulent IGM (e.g. reconnection, resonance with small scale Alfvén waves, etc) may allow a larger fraction of the energy of cluster–turbulence to become available for the reacceleration of relativistic particles. Further research should quantify these processes.

7   ACKNOWLEDGEMENTS

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